A brief introduction to economics Part I	
Tyler Moore	
Computer Science & Engineering Department, SMU, Dallas, TX	
Lecture 4	
Key notions Preferences Motivation Utility Models Expected utility	Notes
Why again are we studying economics?	
 Economics is a social science Studies behavior of individuals and firms in order to predict outcomes 	
 Models of behavior based on systematic observation Usually cannot run experiments as in bench science, but 	
economics has developed ways to cope with differences inherent to observing the world	
 Economics studies trade-offs between conflicting interests 	
 Recognizes that people operate <i>strategically</i> Have devised ways to model people's interests and decision 	
making	

Notes

	3 / 44
Key notions Preferences Utility Expected utility	Motivation Models
the second se	

Economics is not just about money

- Money helps to reveal preferences
- Money can serve as a common measure for costs and benefits
- As a discipline, economics examines much more than interactions involving money
 - Economics studies trade-offs between conflicting interests
 - Conflicting interests and incentives appear in many circumstances where money never changes hands







Reality



Model

Market

- All models are wrong.
- Some are useful.

Notes

Types of models used in economics

Notes

Analytical models: state plausible assumptions about agent's behavior, then examine the implications

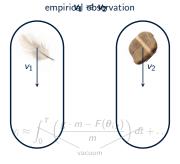
+ Good for theoretical analysis of individual behavior

Motiva

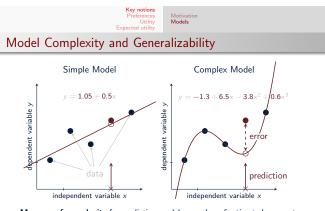
- When models disagree, ground truth can be elusive
- ② Empirical models: observe relationships in aggregate, without explaining underlying individual decisions
 - + Ground truth is achievable
 - Oversimplify, can't explain underlying mechanisms
- **OMEASUREMENT MODELS:** collects data to compare deviations from predictions made by analytical models
 - Directly applying empirical analysis to analytical models usually fails
 - + Offers feedback to analytical models to validate predictions

Motiva Models

Model Complexity and Scientific Discovery



 \rightarrow Drag is part of a complex modelReduction to simple model: drag causes measurement error



Measure of complexity for predictive models: number of estimated parameters

 \rightarrow Risk of $\mathbf{overfitting}$ increases with model complexity

Motivat Models Trade-off on Model Complexity Occam's modeling effort < razor model error number of parameters

 \rightarrow William of Occam († 1349): Principle of model parismony

Notes

6 / 44

7 / 44

8 / 44

Notes

Motivation Utility Expected utility Occam's Razor	Notes
William of Occam, 1285-1349entia non sunt multiplicanda praeter necessitatementities must not be multiplied beyond necessity	
10/44	
Key notions Preferences Utility Expected with Another theory Rational choice theory model Preferences example Our first model: rational choice theory	Notes
 Economics attempts to model the <i>decisions</i> we make, when faced with multiple choices and when interacting with other strategic agents Rational choice theory (RCT): model for decision-making Game theory (GT): extends RCT to model strategic interactions 	
12 / 44	
Key notions Preferences Utility Expected utility Rational choice theory model Preferences example Preferences example	Notes
 Intuitive definition: a rational individual acts in his or her perceived best interest Rationality is what motivates a focus on <i>incentives</i> Question: can you think of scenarios when this definition does not hold in practice? To arrive at a precise definition: use rational choice theory to state available outcomes, articulate preferences among them, 	

Notes

	13 / 44	
Key notions Preferences Utility Expected utility	Rational choice theory model Preferences example	
Model of preferences		

• An agent is faced with a range of possible outcomes $\textit{o}_1,\textit{o}_2 \in \mathcal{O}$, the set of all possible outcomes

- Notation

 - $o_1 \succ o_2$: the agent is strictly prefers o_1 to o_2 . $o_1 \succeq o_2$: the agent weakly prefers o_1 to o_2 ; $o_1 \sim o_2$: the agent is indifferent between o_1 and o_2 ;
- Outcomes can be also viewed as tuples of different properties $\hat{x}, \hat{y} \in \mathcal{O}$, where $\hat{x} = (x_1, x_2, \dots, x_n)$ and $\hat{y} = (y_1, y_2, \dots, y_n)$



Rational choice axioms

Rational choice theory assumes consistency in how outcomes are preferred.

Axiom

Completeness. For each pair of outcomes o_1 and o_2 , exactly one of the following holds: $o_1 \succ o_2$, $o_1 \sim o_2$, or $o_2 \succ o_1$.

 $\Rightarrow\,$ Outcomes can always be compared

Axiom

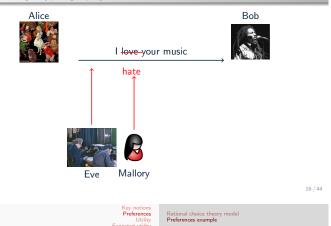
Transitivity. For each triple of outcomes o_1 , o_2 , and o_3 , if $o_1 \succ o_2$ and $o_2 \succ o_3$, then $o_1 \succ o_3$.

 $\Rightarrow\,$ People make choices among many different outcomes in a consistent manner

15 / 44

Preferences Rational choice theory model Utility Preferences example

Example: trade-off between confidentiality and availability using cryptography



Example: trade-off between confidentiality and availability using cryptography

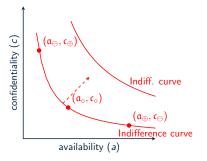
- $\bullet \ \, {\sf Outcomes} \ \, {\cal O}$
 - $\bullet \ \mathfrak{c}_\oplus :$ mechanism achieving high confidentiality
 - \bullet $\mathfrak{c}_\ominus^:$ mechanism achieving low confidentiality
 - $\bullet~\mathfrak{a}_\oplus {:}~\mathsf{mechanism}$ achieving high availability
 - $\bullet \ \mathfrak{a}_\ominus\colon$ mechanism achieving low availability

Preferences

- $\bullet \ \mathfrak{c}_\oplus \succ \mathfrak{c}_\ominus \text{ and } \mathfrak{a}_\oplus \succ \mathfrak{a}_\ominus$
- Taken together: $(\mathfrak{c}_\oplus,\mathfrak{a}_\oplus)\succ(\mathfrak{c}_\ominus,\mathfrak{a}_\ominus)$
- Question: what about high availability and low confidentiality?
- Indifferent: $(\mathfrak{c}_{\oplus}, \mathfrak{a}_{\ominus}) \sim (\mathfrak{c}_{\ominus}, \mathfrak{a}_{\oplus}).$

17 / 44





Notes

Notes

Notes

Key notions Preferences Utility pected utility	Definitions and functions Example

From preferences to utility

- It's great to express preferences, but to make mathematical analysis of decisions possible, we need to transform these preferences into numbers.
- We need a measure of utility, but what does that actually mean?

		20 / 44
Key notions Preferences Utility Expected utility	Definitions and functions Example	

We do not mean utility according to Bentham

- Founder of utilitarianism: "fundamental axiom, it is the greatest happiness of the greatest number that is the measure of right and wrong"
- Utility: preferring "pleasure" over "pain"



Jeremy Bentham

21 / 44



Utility

Rational choice theory defines utility as a way of quantifying consumer preferences

Definition

(Utility function) A utility function U maps a set of outcomes onto real-valued numbers, that is, $U \colon \mathcal{O} \to \mathbb{R}$. U is defined such that $U(o_1) > U(o_2) \iff o_1 \succ o_2$.

Agents make a rational decision by picking the outcome with highest utility:

$$o^* = \arg \max_{o \in \mathcal{O}} U(o) \tag{1}$$

22 / 44 Prefe Definitions and functions erences Utility Example utility functions

IN	otes	

Notes

Notes

- $U(o_1, o_2) = u \cdot o_1 + v \cdot o_2$
 - Useful when outcomes are *substitutes* • Example substitutes: processor speed and RAM
- $U(o_1, o_2) = \min\{u \cdot o_1, v \cdot o_2\}$
 - Useful when outcomes are complements
 - Example complements: operating system and third-party software

Preferences Utility Expected utility

Returning to our crypto example

- $\bullet\,$ First, we need a utility function
 - $U(\mathfrak{a}_i,\mathfrak{c}_i) = u \cdot \mathfrak{a}_i + v \cdot \mathfrak{c}_i$
 - Question: why is this a good choice?
- \bullet For simplicity, we assign $\mathfrak{a}_\oplus=1,\ \mathfrak{a}_\ominus=-1,\ \mathfrak{c}_\oplus=1,$ and $\mathfrak{c}_\ominus=-1$
- $\bullet~$ Utility is in the eye of the beholder
- We consider two scenarios
 - Intelligence agency (u = 1 and v = 3)
 First responders (u = 3 and v = 1)
 - That responders (u = 3 and v = 1)

	24 / 44
Key notions Preferences Utility Expected utility	Definitions and functions Example
Utility of different outcomes	

Outcome	$U_{ m FR}$ (first responder)	U_{intel} (intelligence)
$(\mathfrak{a}_\oplus,\mathfrak{c}_\oplus)$	4	4
$(\mathfrak{a}_\oplus,\mathfrak{c}_\ominus)$	2	-2
$(\mathfrak{a}_{\ominus},\mathfrak{c}_{\oplus})$?	?
$(\mathfrak{a}_\ominus,\mathfrak{c}_\ominus)$?	?

	25 / 44	
Key notions Preferences Utility Expected utility	Definitions Example Attitudes toward risk	
Why isn't utility theory enough?		

- Only rarely do actions people take directly determine outcomes
- Instead there is uncertainty about which outcome will come to pass
- More realistic model: agent selects action *a* from set of all possible actions *A*, and then outcomes *O* are associated with probability distribution



Definition

(Lottery) A lottery is a mapping from all outcomes $(o_1, o_2, \ldots, o_n) \in \mathcal{O}$ to probabilities corresponding to each outcome (p_1, p_2, \ldots, p_n) , where $\sum_{1}^{n} p_i = 1$. A lottery l_1 is represented as $l_1 = \langle o_1 : p_1, o_2 : p_2, \ldots, o_n : p_n \rangle$.

Notes

Notes

Notes



Notes

Notes

- Indeterminism in nature
- Lack of knowledge
- Incompleteness in the model
- Uncertainty concerns which outcome will occur \Rightarrow Known unknowns, NOT unknown unknowns

29 / 44 Definitions Expected utility Expected utility

Definition

(Expected utility (discrete)) The expected utility of an action $a \in \mathcal{A}$ is defined by adding up the utility for all outcomes weighed by their probability of occurrence:

$$E[U(a)] = \sum_{o \in \mathcal{O}} U(o) \cdot P(o|a)$$
(2)

Agents make a rational decision by maximizing expected utility:

$$a^* = \arg\max_{a \in \mathcal{A}} E[U(a)] \tag{3}$$

30 / 44



Figure 2.1: Example exposure time-map with red marking systems with known exploits Source: http://www.cl.cam.ac.uk/~fms27/papers/2011-Leverett-industrial.pdf

Definition Example Expected utility Example: process control system security

Notes

Notes

- Actions available: $\mathcal{A} = \{ \text{disconnect}, \text{connect} \}$
- Outcomes available: $\mathcal{O} = \{ attack, no attack \}$
- If systems are connected, then the probability of successful attack is 0.01 (P(attack|connect) = 0.01)
- If systems are disconnected, then P(attack|disconnect) = 0

31 / 44



Example: process control system security

Action	U	$_{P(\mathrm{attack} \mathrm{action})}$	U	no attack P(no attack action)	E[U(action)]
disconnect	-100	0	5	1	?
connect	-100	0.01	10	0.99	?

$$\begin{split} E[U(a)] &= \sum_{o \in \mathcal{O}} U(o) \cdot P(o|a) \\ E[U(\text{disconnect})] &= U(\text{attack}) \cdot P(\text{attack}|\text{disconnect}) \\ &+ U(\text{no attack}) \cdot P(\text{no attack}|\text{disconnect}) \\ &= -100 \cdot 0 + 5 \cdot 1 \\ &= 5 \end{split}$$

33 / 44

34 / 44

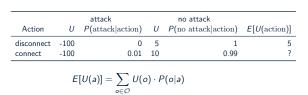
Notes

Notes

Notes

Preferences Utility Expected utility

Example: process control system security



 $E[U(\text{connect})] = U(\text{attack}) \cdot P(\text{attack}|\text{connect})$ $+ U(\text{no attack}) \cdot P(\text{no attack}|\text{connect})$ $= -100 \cdot 0.01 + 10 \cdot 0.99$ = 8.9

 \Rightarrow risk-neutral IT security manager chooses to connect since E[U(connect)] > E[U(disconnect)].

	Key notions Preferences Utility Expected utility	Definitions Example Attitudes toward risk
Let's make a deal		

- Option 1: Take \$10
- Option 2: Get \$20 with a 50% chance, \$0 otherwise
- Which would you choose?
- E[U] = 0.5 * \$20 + 0.5 * \$0 = \$10
- Prefer option 1: you're risk-averse
- Prefer option 2: you're risk-seeking
- Are you indifferent? If so-you're risk-neutral

35/44

 Key notions Preferences Utility Expected utility
 Definitions Example Attitudes toward risk

 Let's make a deal (round 2)

- Option 1: Take \$10
- Option 2: Get \$150 with a 10% chance, \$0 otherwise
- Which would you choose?
- E[U] = 0.1 * \$150 + 0.5 * \$0 = \$15
- Prefer option 1: you're risk-averse
- Prefer option 2: you're risk-neutral or seeking

Example Attitudes toward risk

Let's make a deal (round 3)

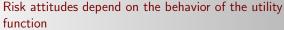
Notes

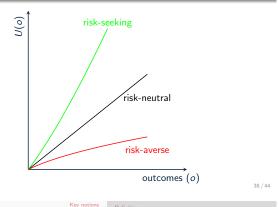
- Option 1: Take \$10
- \bullet Option 2: Get \$50 with a 10% chance, \$0 otherwise
- Which would you choose?
- E[U] = 0.1 * \$50 + 0.5 * \$0 = \$5
- Prefer option 1: you're risk-averse or risk-neutral

Expected utili

• Prefer option 2: you've got a gambling problem

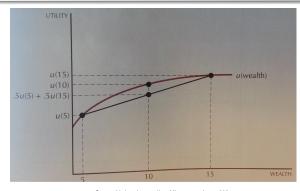






Preferences Utility Expected utility Expected utility

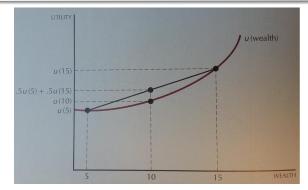
Risk-averse prefer utility of expected value over lottery



Source: Varian, Intermediate Microeconomics, p. 225



Risk-seekers prefer lottery over utility of expected value





39 / 44

Notes

37 / 44

Notes

From attitudes to utility

Notes

Notes

Notes

Notes

- $\bullet\,$ Suppose that outcomes are numeric $\mathcal{O}\in\mathbb{R}$
- When might that happen?
- Then we can define risk-attitudes by how the utility function behaves

Definition			
(Risk neutrality) An a function on <i>o</i> .	agent is risk-	neutral when $U(o)$ is a linear	
			41/44
	Key notions Preferences Utility Expected utility	Definitions Example Attitudes toward risk	
From attitudes to u	utility		

Definition

(Risk aversion) An agent is risk-averse when U(o) is a concave function (i.e., U''(x) < 0 for a twice-differentiable function).

Definition

(Risk seeking) An agent is risk-seeking when U(o) is a convex function (i.e., U''(x) > 0 for a twice-differentiable function).

		42
Key notions Preferences Utility Expected utility	Definitions Example Attitudes toward risk	

Example: antivirus software

- Suppose you have \$10,000 in wealth. You have the option to buy antivirus software for \$75.
- Outcomes available:

 $\mathcal{O} = \{ \text{hacked (decreases wealth by $2,000)}, \\ \text{not hacked (no change in wealth)} \}$

- Without AV software, probability of being hacked is 0.05 (P(hacked|no antivirus) = 0.05)
- With AV software, probability of being hacked is 0 (*P*(hacked|antivirus) = 0)
- Exercise: compute the expected utility of both buying and not buying AV if you are risk-neutral (so that U(o) = o). Would you buy AV software?

43 / 44

/ 44

Kay notions Definitions Utility Expected utility Example: antivirus software

What if you are risk-averse (so that $U(o) = \sqrt{(o)}$)?

<i>Risk-averse</i> Action	U	hack P(hack action)	U	no hack P(no hack action)	E[U(action)]
buy AV don't buy	$\begin{array}{c} \sqrt{9,925} \\ \sqrt{8,000} \end{array}$	0 0.05	$\sqrt{9,925} \ \sqrt{10,000}$	1 0.95	99.6 99.4

Exercise (on your own): How much would you pay for antivirus software if you were risk-neutral and the probability of getting hacked is 0.1 if you don't have AV installed?